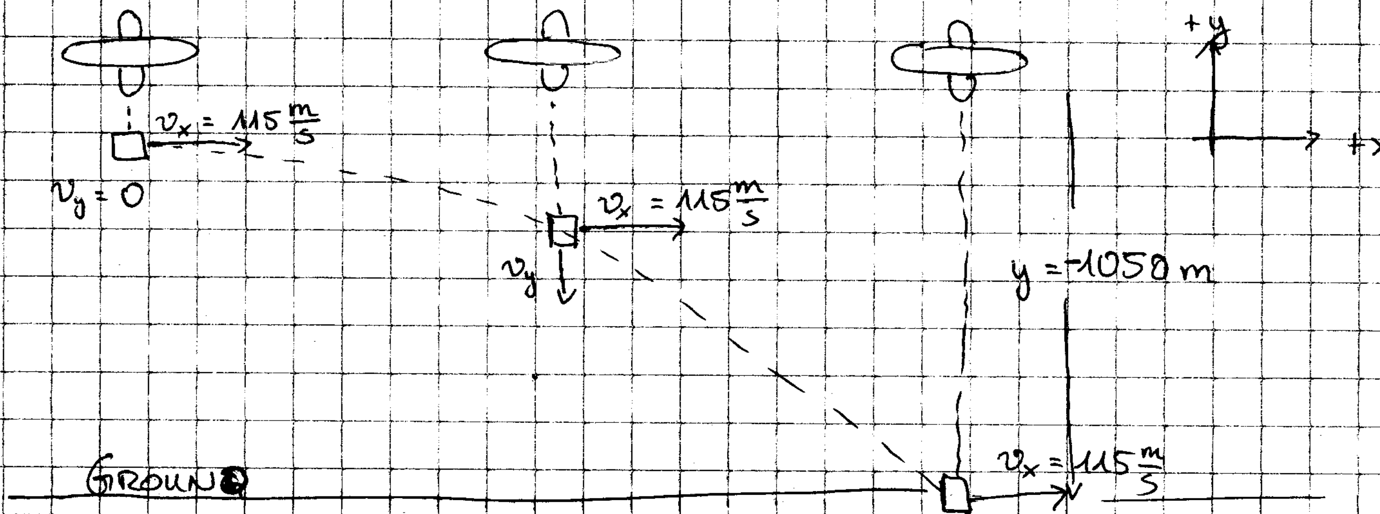
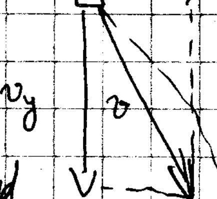
A falling care package — KINEMATICS

An airplane is moving horizontally with a constant velocity $v_0 = 115\,\frac{m}{s}$ at an altitude of $h = 1050\,m$. The plane flies in $+x$ direction and up is defined as $+y$.
The plane releases a "care package" that falls to the ground along a curved trajectory. Ignoring air resistance, determine
a) the time required for the package to hit the ground
b) the ~~speed/velocity~~ and direction of the velocity vector just before it hits the ground.

Drawing (as always) is first:





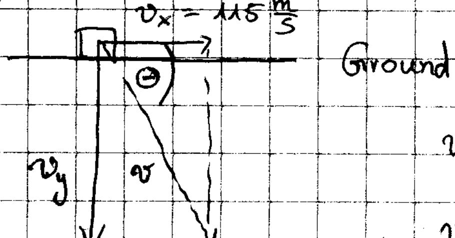
a) horizontal and vertical movement can be treated seperately. Therefore the time it takes for the ~~care~~package to hit the ground is given by the time it takes it to fall.
Falling involves acceleration. The package's initial velocity in y is zero.

$$y = v_{0y}\,t + \frac{1}{2} a_y t^2 \qquad \text{with } a_y = -9.80\,\frac{m}{s^2};\ v_{0y} = 0\,\frac{m}{s} \text{ and } y = -1050\,m$$

$$y = \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-1050\,m)}{-9.80\,\frac{m}{s^2}}} = \underline{\underline{14.6\,s}}$$

b)



$$v = \sqrt{v_x^2 + v_y^2}$$

We need to determine v_x and v_y.

v_x is constant and given as $115\,\frac{m}{s}$.

$$v_y = v_{0y} + a_y t = 0\,\frac{m}{s} + \left(-9.80\,\frac{m}{s^2}\right)\cdot 14.6\,s = -143\,\frac{m}{s}$$

$$\Rightarrow \quad v = \sqrt{\left(115\,\frac{m}{s}\right)^2 + \left(-143\,\frac{m}{s}\right)^2} = 184\,\frac{m}{s}$$

e) final velocity and power ?

An elevator cab of mass $m = 500\,kg$ is descending with speed $v_i = 4.0\,\frac{m}{s}$ when its supporting cable begins to slip, allowing it to fall with constant acceleration $a = \frac{1}{5} g$.

a) During the fall of distance $d = 12\,m$, what is the Work W_g done on the cab by gravitational force F_g ?

b) During the 12 m fall, what is the work W_T done on the cab by the upward pull T (tension in the elevator cable).?

c) What is the net work done on the cab during the fall ?

d) What is the cab's kinetic energy at the end of the 12 m fall ?

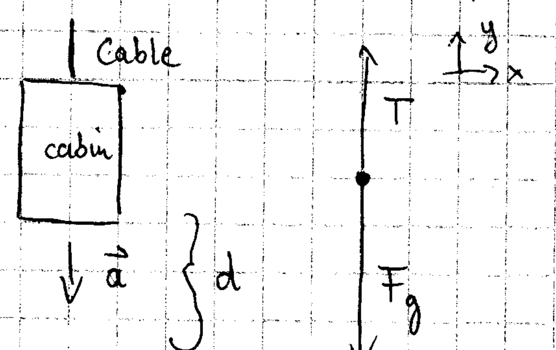
a) We can ~~calculate~~ use $W = F\,d\cos\Theta$ to calculate W_g.

$$W_g = F_g\, d \cdot \underbrace{\cos\Theta}_{\cancel{up}\ down\ \Rightarrow\ \cos\Theta = 1} \qquad / \text{where } F_g = m \cdot g$$

$$W_g = m \cdot g \cdot d \cdot \underbrace{\cos 0^\circ}_{=1} =$$

$$= 500\,kg \cdot 9.80\,\frac{m}{s^2} \cdot 12\,m = 58800\,kg\frac{m^2}{s^2} = \underline{\underline{58.8\,kJ}}$$

b) We can calculate W_T using the same formula $W_T = \cancel{F_T}\, d\cos\Theta$



But we have to find F_T first. We can use Newton's second law along the y axis:

$$\Sigma F_y = m a_y$$

$$\Rightarrow\ T - \underbrace{F_g}_{m \cdot g} = m a_y \quad \Rightarrow T = m(a_y + g)$$

Write as "work" $\quad W_T = T\,d\cos\Theta = m(a_y + g)\,d\cos\Theta$

$$= m\left(-\tfrac{1}{5}g + g\right) d\cos 180^\circ =$$

$$= \frac{4}{5} g\,m\,d\,(-1) = -\frac{4}{5}\left(9.8\,\frac{m}{s^2}\right)(500\,kg)(12\,m) =$$

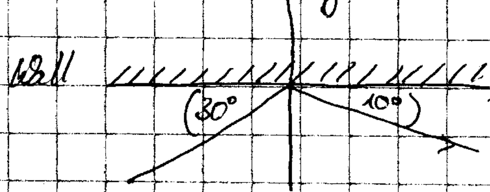
$$= -4.70 \times 10^4\,J$$

c) $W = W_g + W_T = 5.88 \times 10^4\,J - 4.70 \times 10^4\,J = 1.18 \times 10^4\,J \approx 12\,kJ$

d) $EK_f = EK_i + W = \frac{1}{2} m v_i^2 + W = \frac{1}{2}(500\,kg)\left(4.0\,\frac{m}{s}\right)^2 + 1.18 \times 10^4\,J = 1.58 \times 10^4\,J$

A racecar wall collision WORK, ENERGY, POWER, MOMENTUM

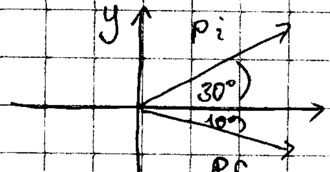
Impuls



A race car bumps into a wall at a 30° angle and leaves the wall at a 10° angle * Just before the collision the car has a velocity of $v_0 = 70\frac{m}{s}$ along a straight line at 30° from the wall. Just after the collision, it is travelling at speed $v_f = 50\frac{m}{s}$ along a straight line at 10° from the wall. The drivers mass is $m = 80$ kg. a) What is ~~his~~ the Impuls J on the driver due to the collision? b) The collision lasts 14 ms. What is the magnitude ~~F~~ of the average force on the driver during the collision?

a) We treat the driver as particle-like (so much less weight than the wall) We cannot calculate J directly ($J = F\Delta t$) since we don't know the force. But we can find J from the change in linear momentum p before and after the collision

$J = p_f - p_i$



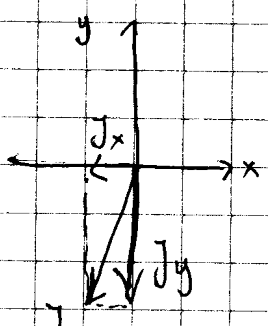
$$J = mv_f - mv_0 = m(v_f - v_0)$$

But we have to take into account the angles:

$$J_x = m(v_{fx} - v_{0x}) = -m(v_{fx}\cos 10° - v_{0x}\cos 30°) =$$

$$= -80\,kg\left(50\frac{m}{s}\cos 10° - 70\frac{m}{s}\cos 30\right) =$$

$$= -910.5\,kg\frac{m}{s}$$



$$J_y = m(v_{fy} + v_{0y}) = 80\,kg\left(-50\frac{m}{s}\sin 10° - 70\frac{m}{s}\sin 30\right) =$$

$$= -3495\,kg\frac{m}{s}$$

$$J = \sqrt{J_x^2 + J_y^2} = \underline{3612}\,kg\frac{m}{s}$$

Direction: $\Theta = \tan^{-1}\frac{J_y}{J_x} = 75.4°$

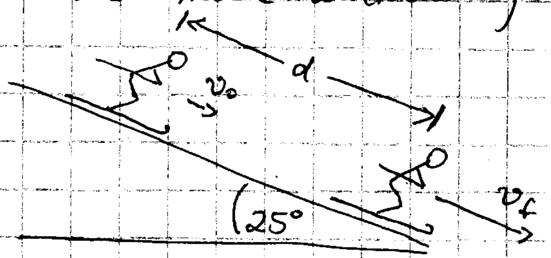
b) $F_{avg} = \frac{J}{\Delta t} = \frac{3612\,kg\frac{m}{s}}{0.014\,s} = 2.58 \times 10^5\,kg\frac{m}{s^2} = \underline{\underline{2.58 \cdot 10^5\,N}}$

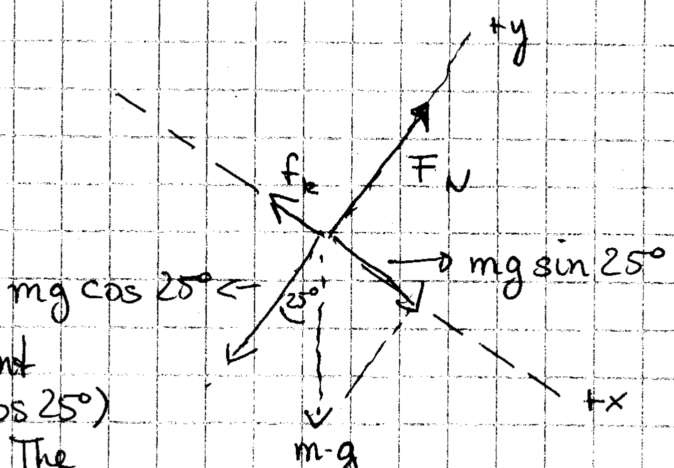
Downhill skiing

ENERGY CONSERVATION

A 58-kg skier is coasting down a 25° slope. A kinetic frictional force of magnitude $f_k = 70\,N$ opposes her motion. Near the top of the slope, the skier's speed is $v_0 = 3.6\,\frac{m}{s}$. Ignoring air resistance, determine the speed v_f at a point that is displaced 57 m downhill.
How is the speed 171 m down the hill and why?

We can use the work-energy theorem to find the final speed. So, we need the work done by the net external force. Let's make a drawing and a free-body diagram.





The normal force F_N is balanced by the component of the skiers weight (mg cos 25°) perpendicular to the slope. The net force points in x direction; that's where the acceleration occurs.

$$\Sigma F_x = mg \cdot \sin 25° - f_k = (58\,kg)(9.80\,\tfrac{m}{s^2})\sin 25° - 70\,N = \underline{\underline{170\,N}}$$

Work is defined as: $W = (\Sigma F_x \cos\theta)\,d =$

$$= (170\,N \cos 0°)(57\,m) = \underline{\underline{9700\,J}}$$

where θ is angle between f_k and direction of movement

Work - Energy theorem: $W = KE_f - KE_0$

We are looking for final speed; so $KE_f = W + KE_0 = W + \frac{1}{2}mv_0^2$

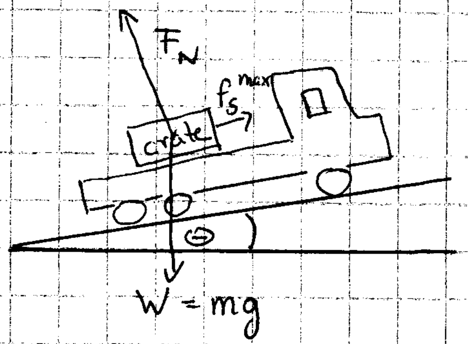
$$= 9700\,J + \tfrac{1}{2}(58\,kg)(3.6\,\tfrac{m}{s})^2 =$$

$$= 10100\,J$$

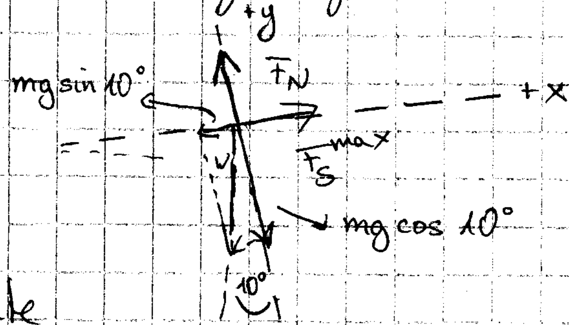
$$KE_f = \frac{1}{2}mv_f^2 \Rightarrow v_f = \sqrt{\frac{2\,KE_f}{m}} = \sqrt{\frac{2(10100\,J)}{58\,kg}} = \underline{\underline{19\,\tfrac{m}{s}}}$$

Hauling a crate — DYNAMICS

A flatbed truck is carrying a crate up a 10° hill. The coefficient of static friction between the truck bed and the crate is $\mu_s = 0.350$. Find the maximum acceleration that the truck can attain before the crate begins to slip backward relative to the truck.



Free body diagram:



Note! We choose the coordinate system ~~th~~ to make the calculation easier

The crate will not slip as long as it has the same acceleration as the truck. We need a net force to act on it to accelerate it. Once the acceleration is larger than the max. magnitude of the frictional force $f_s^m = \mu_s F_N$ the crate will slip. So we are looking for a^{max} at this point.
3 forces: $W = mg$ (weight); F_N from truck bed and f_s^{max}.

x-components: $\sum F_x = -mg \sin 10° + \mu_s F_N = ma^{max}$
(using Newton's second law $\sum F_x = ma^{max}$)

$$\Rightarrow \quad a^{max} = \frac{-mg \sin 10° + \mu_s F_N}{m}$$

We need the value for F_N to calculate a^{max}. We can get F_N from looking at the y components

$$\sum F_y = -mg \cos 10° + F_N = 0 \quad \Rightarrow \quad F_N = mg \cos 10°$$

$$a^{max} = \frac{-\cancel{m}g \sin 10° + \mu_s \cancel{m}g \cos 10°}{\cancel{m}} = g(-\sin 10° + \mu_s \cos 10°) =$$

$$= 9.80\,\frac{m}{s^2}\,(\sin 10° + 0.350 \cos 10°) = 1.68\,\frac{m}{s^2}$$