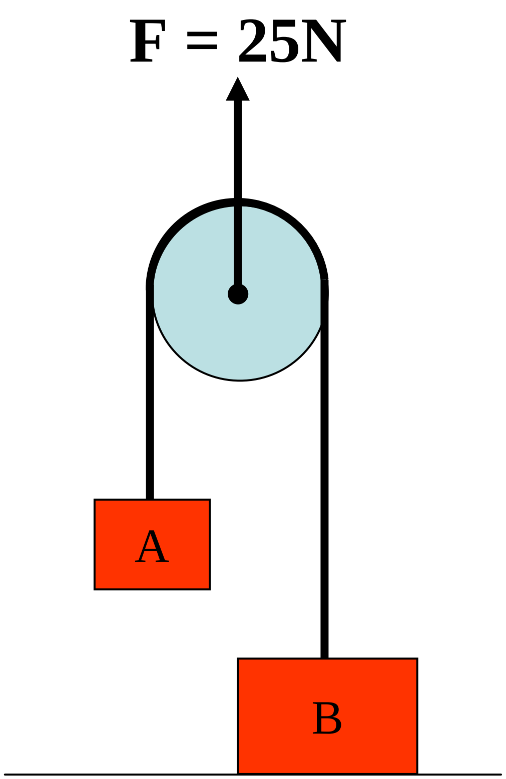
Free-Body Diagrams: Summary of Important Concepts

- **For each object, draw a free-body diagram, showing all the forces acting *on* the object. Do not include any forces which do not act on the object. For example a person pulling, does not act on the person, it acts on whatever he is pulling.**
- **If there are multiple objects, draw a separate diagram for each one.**
- **Use Newton's laws to describe the motion for each object, by itself. Treat orthogonal directions as being independent.**

Example Problem



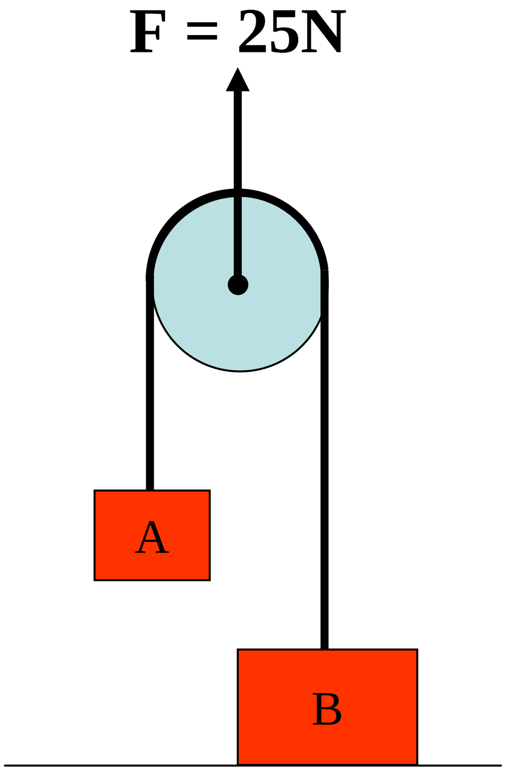
$M_A = 1.5$ Kg

$M_B = 2.5$ Kg

Two masses are attached by a cord, as shown. Mass B is resting on the ground. An upward force of 25N is applied to the pulley. Assume massless cord and pulley, and no friction.

- What is the tension in the cord?
- What are the accelerations of each mass?
- With what force would one need to pull to lift B off the ground?

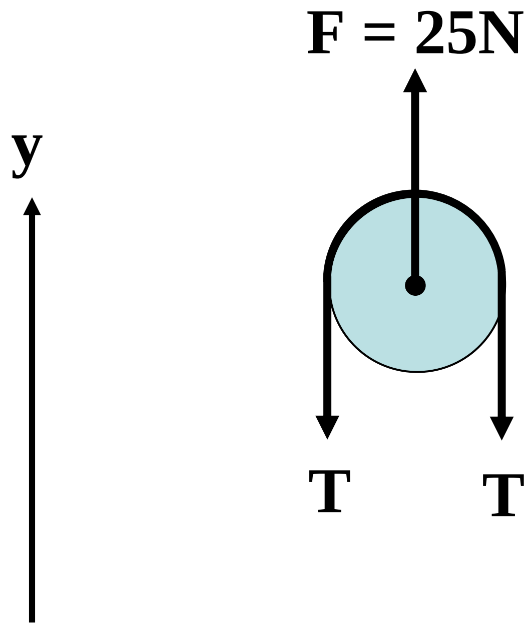
Example Problem



$M_A = 1.5$ Kg

$M_B = 2.5$ Kg

- What is the tension in the cord?

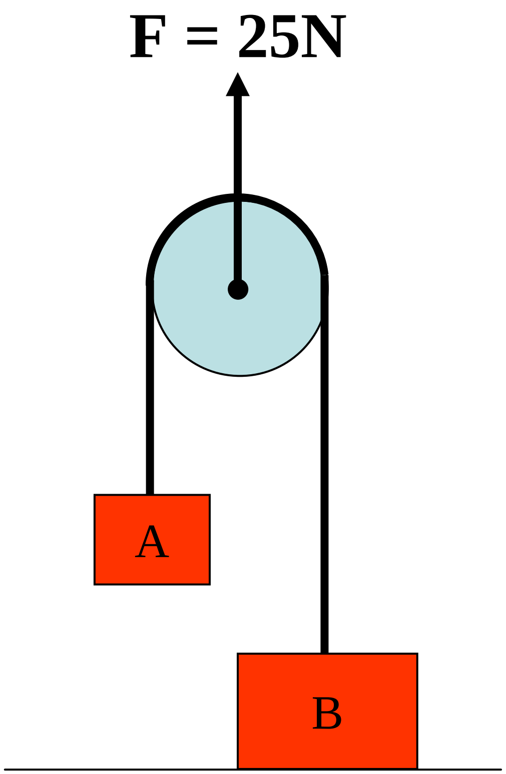


$$\sum F_y = m_p a = 0$$

$$\left(m_p = 0\right)$$

$$F - 2T = 0$$

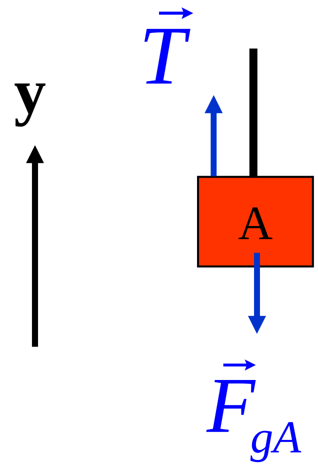
$$T = 12.5N$$

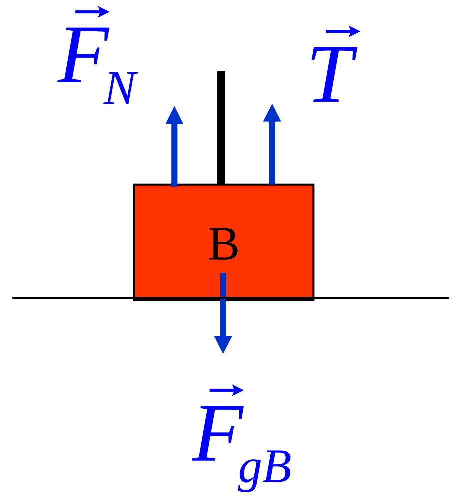


$M_A = 1.5$ Kg

$M_B = 2.5$ Kg

- What are the accelerations of each mass?





$$\sum F_y = M_A a$$

$$T - M_A g = M_A a$$

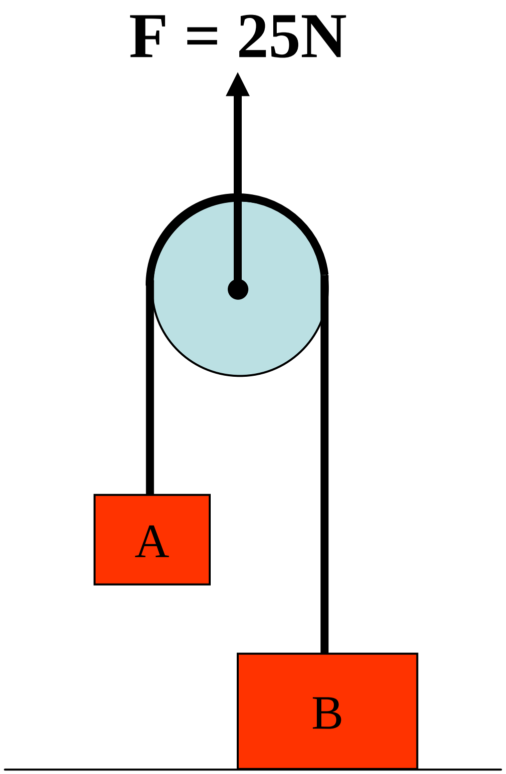
$$12.5 - 1.5(9.8) = 1.5a$$

$$a = -1.47 \text{ m/s}^2$$

(down)

Downward force of gravity more than tension, so it stays on floor.

$$a = 0$$

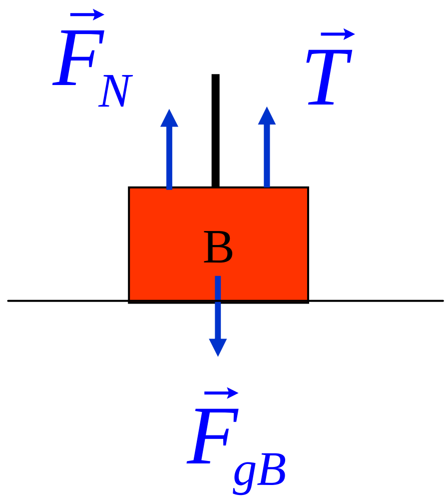


$M_A = 1.5$ Kg

$M_B = 2.5$ Kg

- With what force would one need to pull to lift B off the ground?

Requires tension greater than weight.



$$T > M_B g$$

$$F = 2T$$

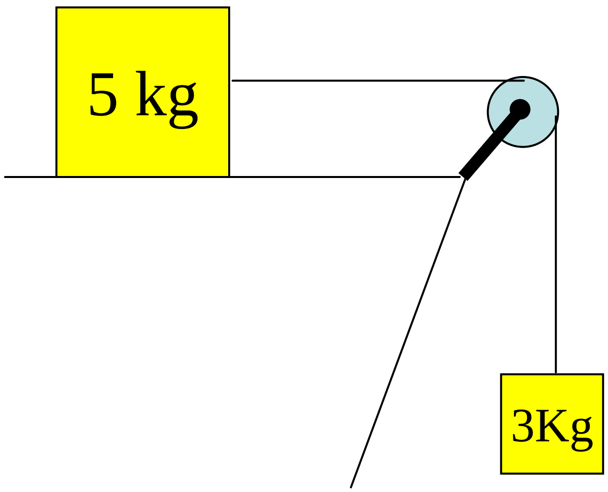
$$F > 2M_B g$$

$$F > 49 \text{ N}$$

Example Problems

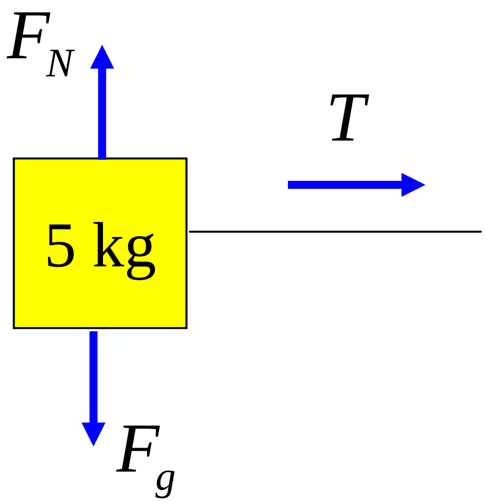
In the setup below, assume the pulley is massless and ignore friction.

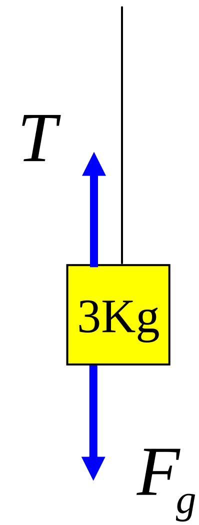
How far does the 3 Kg mass fall in first 2 seconds after being released from rest?



Since they are joined, the acceleration of each will be the same.

Calculate acceleration:





$$\sum F_x = 5a$$

$$T = 5a$$

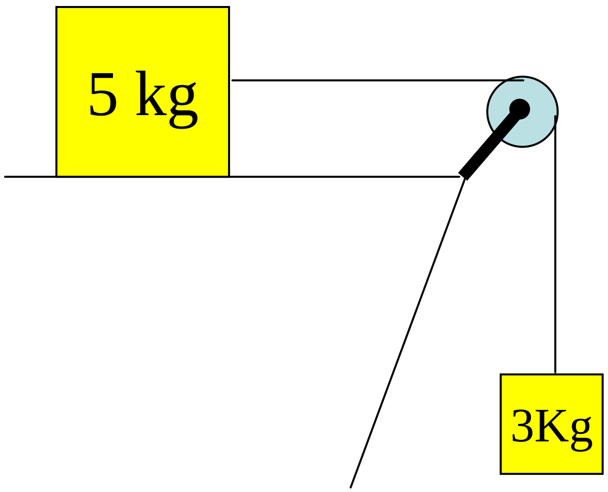
$$\sum F_y = 3a$$

$$-T + 3g = 3a$$

$$-5a + 3g = 3a$$

$$a = \tfrac{3}{8} g = 3.67 \text{ m/s}^2$$

How far does the 3 Kg mass fall in first 2 seconds after being released from rest?

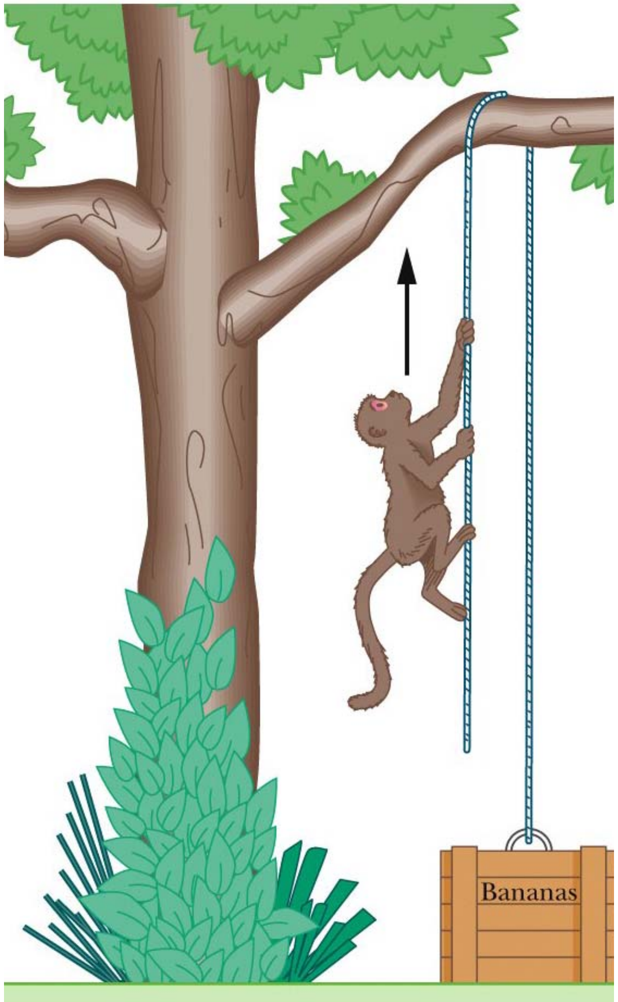


$$y - y_0 = v_i t + \tfrac{1}{2} a t^2$$

$$\Rightarrow \ \Delta y = \tfrac{1}{2} a t^2$$

$$\Delta y = \tfrac{1}{2}(3.67)2^2 = 7.34 \text{ m}$$

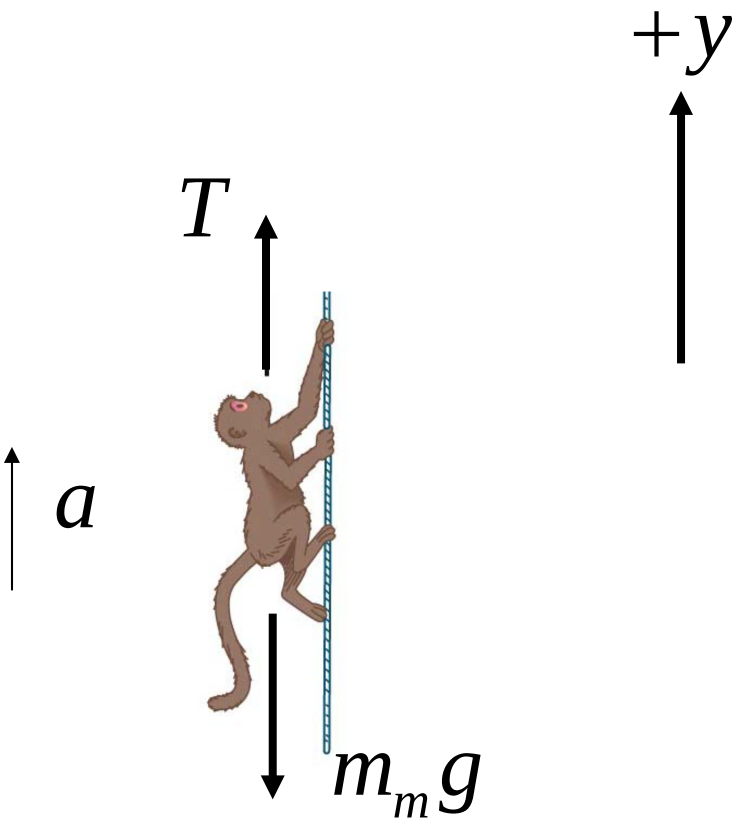
Example Problem (From text)

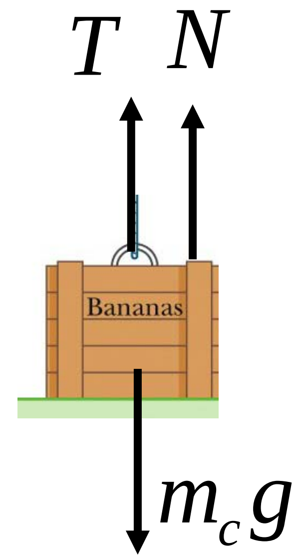


A 10 Kg monkey climes up a massless/frictionless rope. The crate weighs 15 Kg.

a) What is the least acceleration the monkey must have in order to just lift the crate.

b) If the monkey later stops, what will be the monkeys acceleration and tension in the rope?



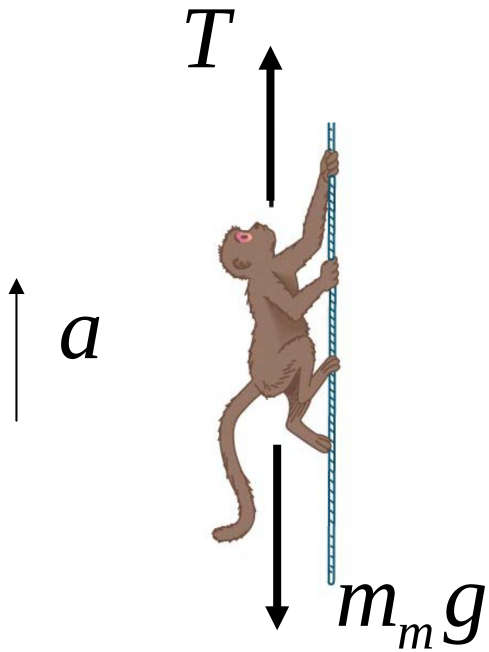


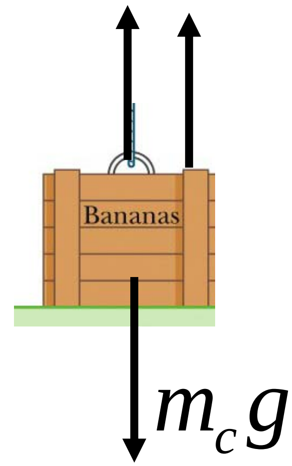
$$\sum F_y = m_c a = T + N - m_c g$$

$$\sum F_y = m_m a = T - m_m g$$

Free Body Diagrams

If crate "just lifts off" this means Normal force = 0 or T=mg





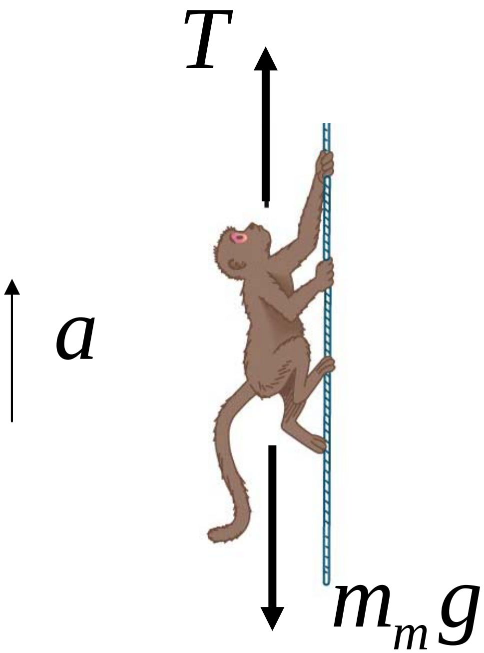
$$T = m_c g$$

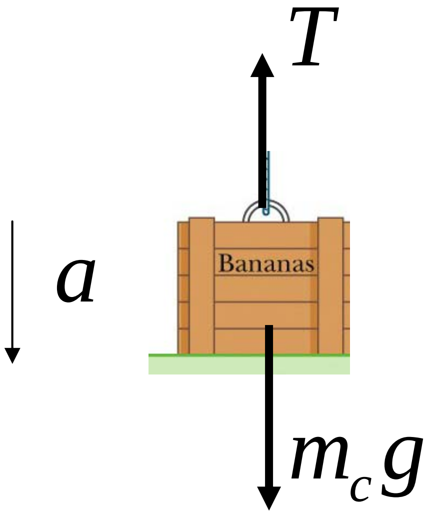
$$m_m a = T - m_m g$$

$$a = \frac{T - m_m g}{m_m} = \frac{m_c g - m_m g}{m_m}$$

$$a = \frac{15-10}{10} g = 4.9 \ \mathrm{m}/s^2$$

Afterwards: Monkey will accelerate up while crate falls. Both will have same magnitude.





$$m_m a = T - m_m g \quad ①$$

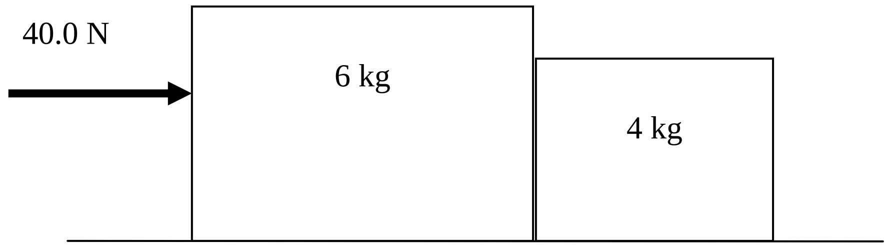
$$-m_c a = T - m_c g \quad ②$$

$$①\text{-}② \quad \rightarrow (m_m + m_c)a = (m_c - m_m)g$$

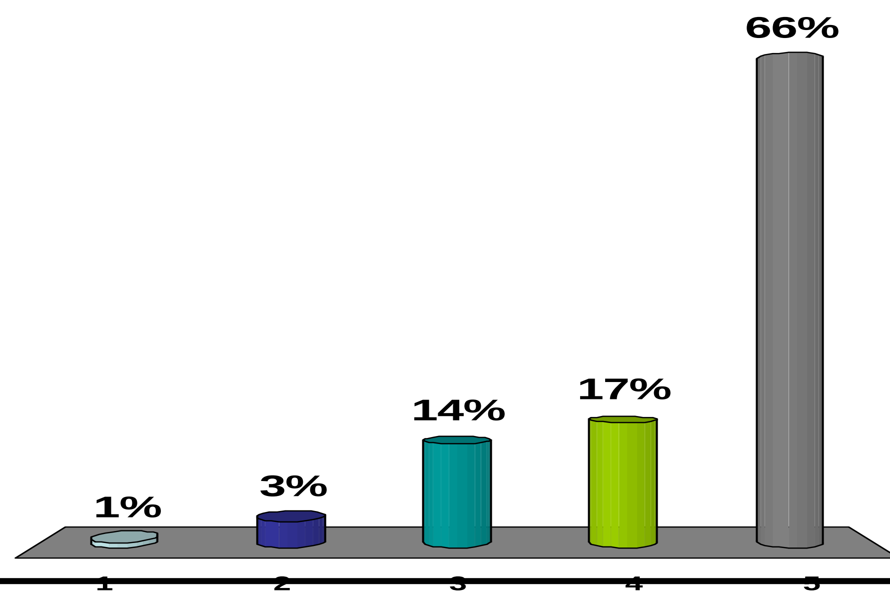
$$a = 0.2g = 1.96 \text{ m/s}^2$$

A few Multiple Choice Questions from Last Years First Midterm

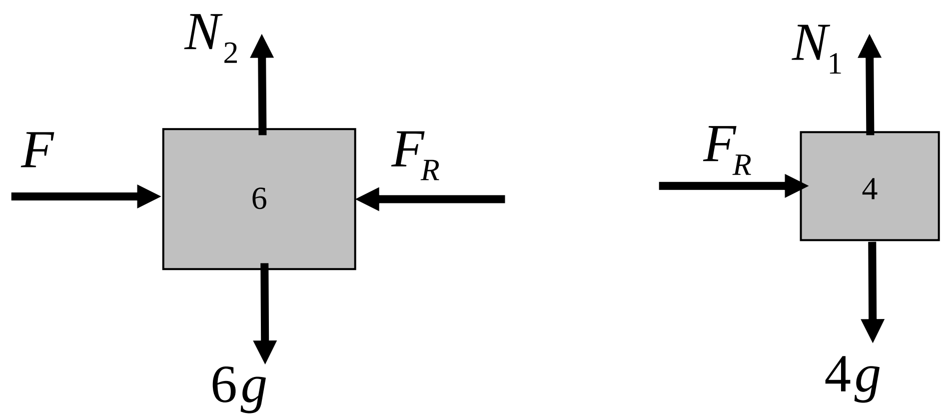
A 6.0 kg block is in contact with a 4.0 kg block on a frictionless surface as shown. The 6.00 kg block is being pushed by a force of 40.0 N toward the 4.0 kg block. What is the magnitude of the force of the 6.0 kg block on the 4.0 kg block?



1. 8 N
2. 20 N
3. 16 N
4. 24 N
5. 40 N



Intuitively, a force of 40 N is applied to the total mass of 10 kg. Hence the acceleration of the two masses will be 4 m/s2. Considering only the smaller block, if it has an acceleration of 4 m/s2, and a mass of 4 kg, then the net force acting on it must be 16 N. One could also draw free body diagrams to give:



$$\sum F_x = 6a = F - F_R$$

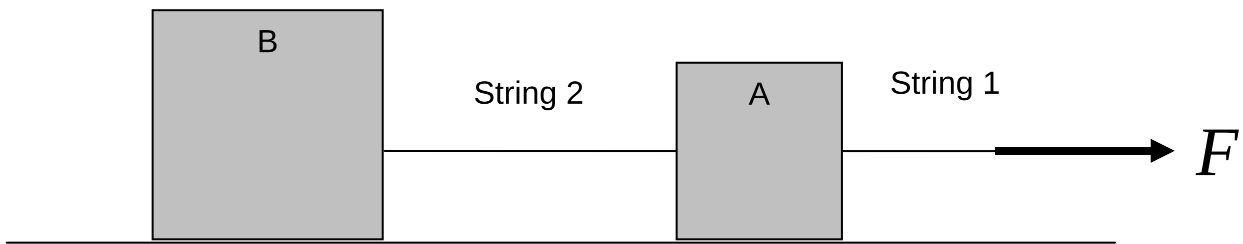
$$\sum F_x = 4a = F_R$$

$$a = F_R / 4$$

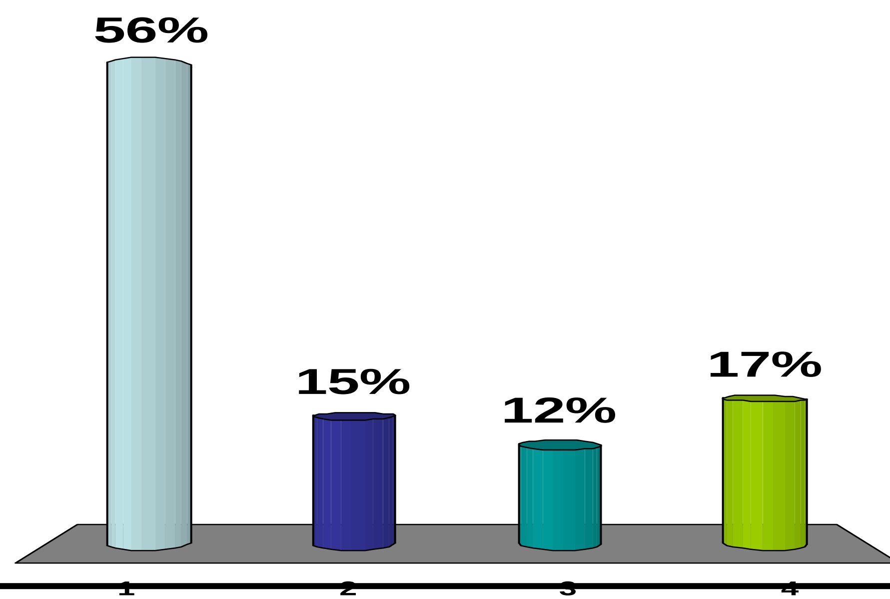
$$6\frac{F_R}{4} = 40 - F_R$$

$$F_R = 16 \text{ N}$$

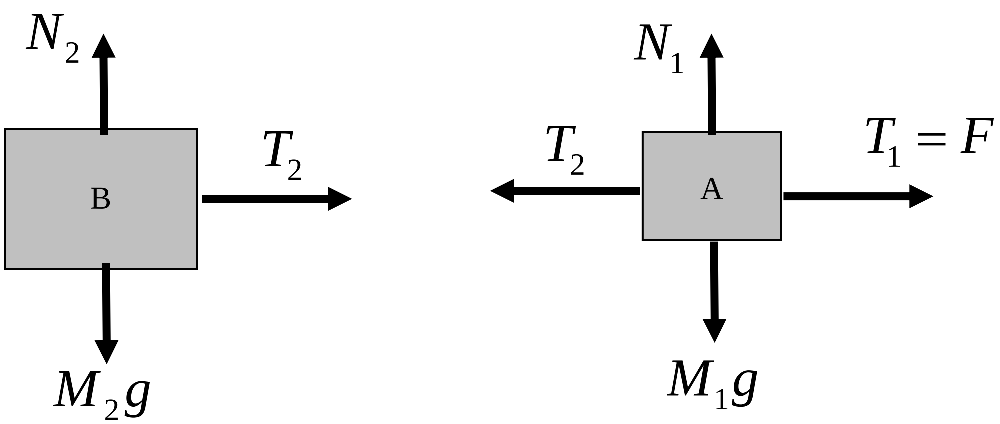
In the figure below, the two blocks have non-zero mass. The strings can be considered massless and friction can be ignored. Block A is pulled with a constant non-zero force *F*. How does the magnitude of the tension in string 1, T_1 compare with the tension in string 2, T_2?



1. $T_1 > T_2$ ✓
2. $T_1 = T_2$
3. $T_1 < T_2$
4. It depends on the relative masses of A and B



This can be answered intuitively, or by using free body diagrams. Since the system is frictionless, with a constant force there will be a constant acceleration, which is the same for each block as they are attached. String 1 has a tension due to the combined mass of A and B, String 2 has only the mass of B, so the tension T_2 is always less than T_1

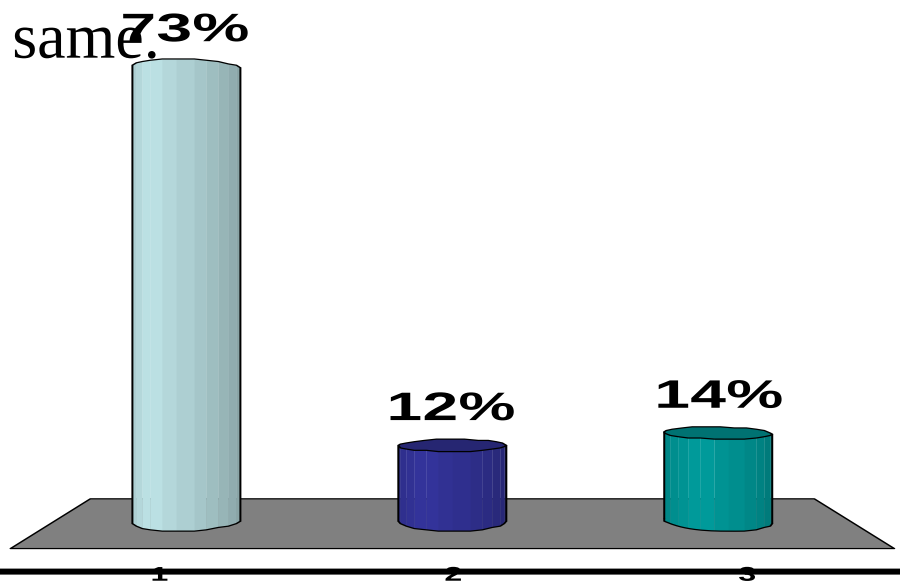


$$\sum F_x = m_B a = T_2 \qquad \sum F_x = m_A a = T_1 - T_2$$

$$T_1 - T_2 = m_A \frac{T_2}{m_B} \qquad T_1 = \left(1 + \frac{m_A}{m_B}\right) T_2$$

You wish to pry open the door of a safe acquired during a bank heist using two horses. To get the most tension acting on the door, you:

1. Tie two horses to the door of the safe, and tie the rear of the safe to a tree, and have the two horses pull in the same direction.
2. Tie one horse to the front of the safe and one horse to the rear, and have them pull in opposite directions.
3. Both *A*. and *B*. above have a "force obtainable from two horses", and are therefore the same.




HAPPY THANKSGIVING